THEORY OF THE INTERACTION OF A SHOCK WAVE WITH A BOUNDARY LAYER ON A MOVING SURFACE

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The behavior of a boundary layer in the vicinity of a separation point on a surface, which is at rest or moves with a velocity $u_w = O(\mathrm{Re}^{-1/8})$ (Re is the Reynolds number), is described by the "triple-decker" flow model first introduced by Krapivskii et al. [1, 2]. In the case where the surface moves downstream with a velocity $\mathrm{Re}^{-1/8} \ll u_w \ll 1$, the problem has been solved for supersonic flows [3, 4] and for an incompressible fluid [5]. Zhuk [6] has published numerical results within the framework of the triple-decker scheme for a surface moving upstream.

In the present article we investigate the motion of a surface upstream with a velocity $\mathrm{Re}^{-1/8} \ll u_w \ll 1$. We assume that the flow is described by the Navier–Stokes equations, the external (freestream) flow is supersonic, its parameters have been reduced to dimensionless form by the usual convention, and $\mathrm{Re} \to \infty$. We first obtain a solution of the linearized problem by the triple-decker scheme and pass to the limit $u_w \mathrm{Re}^{1/8} \to \infty$; we then analyze the problem for $\mathrm{Re}^{-1/8} \ll u_w \ll 1$.

1. In the triple-decker model, as we know, the flow is partitioned into three zones: an external "inviscid" flow with thickness $y = O(\mathrm{Re}^{-3/8})$, the main part of the boundary layer $y = O(\mathrm{Re}^{-1/2})$, and the wall layer $y = O(\mathrm{Re}^{-5/8})$, which provides the main contribution to the induced pressure. The length of the interaction zone is $O(\mathrm{Re}^{-3/8})$, and the characteristic differential pressure is $O(\mathrm{Re}^{-1/4})$. If the flow parameters are reduced to dimensionless form as in [2], the equations describing the flow in the wall layer have the form

$$U\partial U/\partial x + V\partial U/\partial y + P'(x) = \partial^2 U/\partial y^2, \quad \partial U/\partial x + \partial V/\partial y = 0, \tag{1.1}$$

where U and V are the longitudinal and transverse components of the velocity vector, respectively, and P is the pressure. The no-slip condition holds at the wall, and the disturbances decay to zero in the limit $x \to \pm\infty$:

$$U = -U_0, \quad V = 0 \quad \text{at } y = 0; \quad \lim_{x\to\pm\infty} U = y - U_0. \tag{1.2}$$

The pressure induced by the finite displacement thickness is given by the Ackeret equation:

$$\lim_{y\to+\infty} (U - y) = A(x), \quad -A'(x) = P - h\theta(x). \tag{1.3}$$

Here U_0 is the negative of the dimensionless velocity at the wall, and h is the differential pressure across the discontinuity. The problem can be solved completely in the linear approximation, i.e., in the limit $h \to 0$. The expansion $U = y - U_0 + hU^{(1)} + \ldots$, $V = hV^{(1)} + \ldots$, $P = hP^{(1)} + \ldots$ is valid in this case. Substituting it in Eq. (1.1), we obtain

$$(y - U_0)\partial U^{(1)}/\partial x + V^{(1)} + P^{(1)\prime}(x) = \partial U^{(1)}/\partial y^2, \quad \partial U^{(1)}/\partial x + \partial V^{(1)}/\partial y = 0. \tag{1.4}$$

The boundary conditions (1.2) and (1.3) acquire the form

$$U^{(1)}(x, 0) = V^{(1)}(x, 0) = U^{(1)}(\pm\infty, y) = 0; \tag{1.5}$$

$$(\partial U^{(1)}/\partial x)(x, +\infty) = \theta(x) - P^{(1)}(x). \tag{1.6}$$

The Fourier transform with respect to x reduces problem (1.4)-(1.6) to a system of ordinary differential equations. We solve it to obtain

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$$U^{(1)} = \int_0^y dy \int_{-\infty}^{+\infty} e^{ikx} \frac{(ik)^{-1/3} Ai\left((ik)^{1/3}(y - U_0)\right)}{\left\{Ai'\left(-(ik)^{1/3}U_0\right) + (ik)^{4/3} \int\limits_{-(ik)^{1/3}U_0}^{+\infty} Ai(z)\,dz\right\}} \frac{dk}{2\pi}; \quad (1.7)$$

$$P^{(1)} = \int_{-\infty}^{+\infty} e^{ikx} \frac{Ai'\left(-(ik)^{1/3}U_0\right)}{\left\{Ai'\left(-(ik)^{1/3}U_0\right) + (ik)^{4/3} \int\limits_{-(ik)^{1/3}U_0}^{+\infty} Ai(z)\,dz\right\}} \frac{dk}{2\pi i\,(k - i0)}, \quad (1.8)$$

where $|\arg(ik)| < \pi$, and $(ik)^{1/3} > 0$ for $(ik) > 0$. At $x < 0$ and for $\operatorname{Im} k \leq 0$ the integrands are regular (except at a finite set of poles) and vanish in the limit $k \to \infty$. Consequently, the behavior of Eqs. (1.7) and (1.8) at negative x is determined by the integrand singularities, which coincide (for $\operatorname{Im} k < 0$) with the zeros of the expression in the braces. It has been shown [7] that these zeros are situated along the negative imaginary semiaxis, the number of them is finite [$O(U_0{}^{8/5})$ in the limit $U^0 \to \infty$), and they correspond to the eigenfunctions of problem (1.4)-(1.6), which have the form

$$U^{(1)} \sim e^{(ik)_* x} \int_0^y Ai\left((ik)_*^{1/3}(y - U_0)\right) dy, \quad P^{(1)} \sim e^{(ik)_* x},$$

where $(ik)_*$ is a certain zero of the expression in the braces, which we have already noted is real and positive. If the velocity perturbations did not obey the extinction condition in the limit $x \to +\infty$, the solution of the problem would not be unique and would differ from (1.7), (1.8) by an arbitrary linear combination of these eigenfunctions.

We now analyze the case $U \to +\infty$ in more detail. We write the expression in the braces as

$$\left\{Ai'\left(-(ik)^{1/3}U_0\right) + (ik)^{4/3} \int\limits_{-(ik)^{1/3}U_0}^{+\infty} Ai(z)\,dz\right\} = Ai'\left(-(ik)^{1/3}U_0\right) \times$$

$$\times \left[1 + ikU_0^{-1} + (-ikU_0^{-1})\, g\left(-(ik)^{1/3}U_0\right)\right], \quad g(\xi) = 1 + \xi \int_\xi^{+\infty} Ai(z)\,dz/(Ai'(\xi)).$$

We invoke the asymptotic representation of $Ai(z)$ in the limit $z \to \infty$, $|\arg z| < \pi$, and we write $g(\xi) = O(\xi^{-1})$ in the limit $\xi \to \infty$, $|\arg \xi| < \pi$; also, $g[-(ik)^{1/3}U_0]$ is bounded for real k, because $Ai'(z)$ vanishes only at negative real z. This implies that

$$\left|ikU_0^{-1} g\left(-(ik)^{1/3}U_0\right)\right| = o\left(\left|1 + ikU_0^{-1}\right|\right) \text{ for } U_0 \to \infty, \quad \operatorname{Im} k = 0$$

and in the principal approximation for $U_0 \to \infty$, Eqs. (1.7) and (1.8) can be represented as

$$U^{(1)} = \int_0^y dy \int_{-\infty}^{+\infty} e^{ikx} \frac{(ik)^{-1/3} Ai\left((ik)^{1/3}(y - U_0)\right)}{Ai'\left(-(ik)^{1/3}U_0\right)\left(1 + (ik)U_0^{-1}\right)} \frac{dk}{2\pi}; \quad (1.7')$$

$$P^{(1)} = \int_{-\infty}^{+\infty} e^{ikx} \frac{dk}{2\pi i\,(k - i0)\left(1 + ikU_0^{-1}\right)}. \quad (1.8')$$

These expressions, in turn, are rewritten in the form of convolutions:

$$U^{(1)} = \int_{-\infty}^{+\infty} \tilde{u}(\xi, y)\, G(x - \xi)\, d\xi; \quad (1.9)$$

$$P^{(1)} = \int_{-\infty}^{+\infty} \tilde{P}(\xi)\, G(x - \xi)\, d\xi, \quad (1.10)$$

where

$$\tilde{u}(x, y) = \int_0^y dy \int_{-\infty}^{+\infty} e^{ikx} (ik)^{-1/3} \frac{Ai\left((ik)^{1/3}(y - U_0)\right)}{Ai'\left(-(ik)^{1/3}U_0\right)} \frac{dk}{2\pi}; \quad (1.11)$$

$$\tilde{P}(x) = \int_{-\infty}^{+\infty} e^{ikx} \frac{dk}{2\pi i (k - i0)} = \theta(x); \tag{1.12}$$

$$G(x) = \int_{-\infty}^{+\infty} e^{ikx} \frac{dk}{2\pi \left(1 + ikU_0^{-1}\right)} = U_0 \theta(x) \exp(-U_0 x). \tag{1.13}$$

It is verified by direct substitution that Eq. (1.11) is a solution of Eqs. (1.4) subject to the boundary conditions (1.5) for the pressure given by Eq. (1.12). Even though problem (1.4), (1.5), (1.12) is parabolic in x, it is still formulated as a boundary-value problem with respect to the longitudinal coordinate.

If we lift the extinction condition in the limit $x \to +\infty$, we find that the solution is not unique and differs from Eq. (1.11) by an arbitrary linear combination of the eigenfunctions of problem (1.4), (1.5), (1.12), which have the form

$$\tilde{u} \sim e^{(ik)_* x} \int_0^y Ai\left((ik)_*^{1/3}(y - U_0)\right) dy,$$

where $(ik)_*$ is a certain root of the equation $Ai'[-(ik)^{1/3}U_0] = 0$.

The scale transformation $\tilde{k} = kU_0^3$, $\tilde{x} = xU_0^{-3}$, $\tilde{y} = yU_0^{-1}$, $\tilde{u} = u^*U_0^{-1}$ eliminates the explicit dependence of (1.11) on U_0; we infer from this fact that the characteristic variation of $\tilde{u}$ takes place in a region $y = O(U_0)$, $x = O(U_0^3)$ (region B in Fig. 1). We also note that $\tilde{u}$ is continuous everywhere except at $x = 0$. At this point $\tilde{u}(+0, y) - \tilde{u}(-0, y) = U_0^{-1}$. This result can be obtained, for example, by integrating Eqs. (1.4) with the pressure (1.12) with respect to x in the neighborhood of $x = 0$.

Since G(x) decays far more rapidly [at $x = O(U_0^{-1})$] than $\tilde{u}(x, y)$, Eq. (1.9) can be written as follows (in the principal approximation for $U_0 \to \infty$) at points where $\tilde{u}$ is continuous:

$$U^{(1)}(x, y) = \tilde{u}(x, y) \int_{-\infty}^{+\infty} G(\xi)\, d\xi = \tilde{u}(x, y).$$

In a small neighborhood of the point of discontinuity $\tilde{u} = \tilde{u}(-0, y)\theta(-x) + \tilde{u}(+0, y)\theta(x) + o(1)$, $x \to 0$, and Eq. (1.9) has the form

$$U^{(1)} = \tilde{u}(-0, y)\left\{\theta(-x) + \theta(x)\, e^{-U_0 x}\right\} + \tilde{u}(+0, y)\,\theta(x)\left\{1 - e^{-U_0 x}\right\}.$$

Equation (1.10) is integrated exactly:

$$P^{(1)} = \theta(x)\left[1 - e^{-U_0 x}\right]. \tag{1.14}$$

Consequently, in the limit $U_0 \to \infty$ the viscous sublayer of the interaction zone decays into two regions: A and B. The longitudinal scales of regions A and B are $O(U_0^{-1})$ and $O(U_0^2)$, respectively. The thickness of the regions is $O(U_0)$. The velocity perturbations decay in region B, and in region A the discontinuity of the longitudinal component of the velocity vector and the pressure is smoothed. If we estimate the contribution of the displacement thickness to the pressure, we find that it is of the order of unity in region A and is $O(U_0^{-4})$ in region B. However, even though the influence of displacement is negligible in region B, it must still be taken into consideration, because the initial velocity profile for region A, in which the interaction problem is now formulated, begins to take shape in the displacement layer. It is also interesting that the pressure perturbations do not propagate upstream away from the discontinuity [Eq. (1.14)]. This result is only asymptotically true in the limit $U_0 \to \infty$. For finite wall velocities the pressure perturbations, as should be expected, propagate in both directions away from the discontinuity.

2. The solution retains a similar structure in the nonlinear case. We assume at the outset that $Re^{-1/8} \ll u_w \ll 1$ and that the differential pressure across the discontinuity $\Delta p = O(u_w^2)$. The condition that the slopes of the streamlines and the differential pressure in the interaction zone are of the same order implies that the characteristic length of the interaction zone $\Delta x = O(Re^{-1/2}u_w^{-1})$.

The length of the region in which the perturbations decay due to viscosity is $O(u_w^3)$. In the linear case these are regions A and B, respectively. Since their thicknesses, by

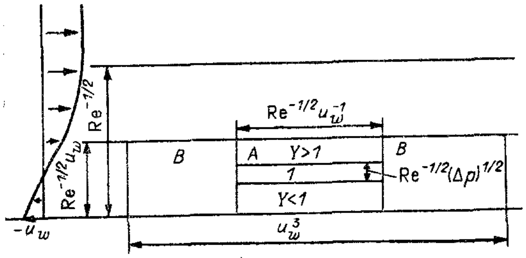

Fig. 1

analogy with [3], are $O[\mathrm{Re}^{-1/2}(\Delta p)^{1/2}] \ll \mathrm{Re}^{-1/2}$, we assume that the viscosity and density are constant and equal to their wall values, and also that the unperturbed velocity profile is linear. We make the following changes of variables:

in region B

$$y = [u'(0)]^{-1}\,\mathrm{Re}^{-1/2}u_w Y, \quad x = (\rho_w/\rho_e)(\mu_e/\mu_w)[u'(0)]^{-2}u_w^3\tilde{x},$$
$$u = u_w\tilde{u}, \quad v = (\mu_w/\mu_e)(\rho_e/\rho_w)u'(0)\,\mathrm{Re}^{-1/2}u_w^{-1}\tilde{v}, \quad p = (\rho_w/\rho_e)u_w^2\tilde{P};$$

in region A

$$y = [u'(0)]^{-1}\,\mathrm{Re}^{-1/2}u_w Y, \quad x = (\rho_e/\rho_w)[u'(0)]^{-1}(M_e^2-1)^{-1/2}\,\mathrm{Re}^{-1/2}u_w^{-1}X,$$
$$u = u_w\tilde{U}, \quad v = (\rho_w/\rho_e)(M_e^2-1)^{1/2}u_w^3\tilde{V}, \quad p = (\rho_w/\rho_e)u_w^2\tilde{P}.$$

Here $u'(0)$ is the slope of the unperturbed velocity profile at the wall in the scale of the boundary layer; the indices w and e refer to the values of the variables at the wall and in the external flow. We also assume that the differential pressure across the discontinuity is represented by the equation $\tilde{P} = h\theta(x)$ [$h = (\rho_e/\rho_w)(\Delta p/u_w^2)$ is the dimensionless intensity of the discontinuity]. The equations describing the flow then acquire the form:

in region B

$$\tilde{u}\partial\tilde{u}/\partial\tilde{x} + \tilde{v}\partial\tilde{u}/\partial Y + h\delta(\tilde{x}) = \partial^2\tilde{u}/\partial Y^2, \ \partial\tilde{u}/\partial\tilde{x} + \partial\tilde{v}/\partial Y = 0,$$
$$\tilde{u}(\tilde{x}, 0) = -1, \ \tilde{v}(\tilde{x}, 0) = 0, \ (\partial\tilde{u}/\partial Y)(\tilde{x}, +\infty) = 1, \ \tilde{u}(\pm\infty, Y) = Y - 1; \tag{2.1}$$

in region A

$$\tilde{U}\partial\tilde{U}/\partial X + \tilde{V}\partial\tilde{U}/\partial Y + \tilde{P}(X) = 0, \ \partial\tilde{U}/\partial X + \partial\tilde{V}/\partial Y = 0,$$
$$\tilde{V}(X, 0) = 0, \quad \lim_{Y\to+\infty}(\tilde{U} - Y) = A(X), \quad A'(X) = h\theta(X) - \tilde{P}. \tag{2.2}$$

The initial condition for $\tilde{U}$ is determined from the conditions for matching of the solutions in regions A and B:

$$\lim_{X\to\pm\infty}\tilde{U}(X, Y) = \lim_{\tilde{x}\to\pm 0}\tilde{u}(\tilde{x}, Y), \tag{2.3}$$

where it is sufficient to specify only one limit, either $X \to +\infty$ or $X \to -\infty$. It is readily apparent that the second limit holds automatically. Condition (2.3) distinguishes this case from the case discussed in [3, 4], where the wall moves downstream. In [3, 4] the unperturbed velocity profile was taken as the initial profile for the interaction zone. It is also interesting that the problem is formulated as a boundary-value problem in the longitudinal coordinate, even though the flow in region B is described by ordinary boundary-layer equations. Such a formulation follows from the linear analysis [problem (1.4), (1.5), (1.12) in Sec. 1 above], where it was shown that the linearized problem (2.1) has a denumerable set of eigenfunctions that grow exponentially downstream. The extinction conditions for the velocity perturbations in the limit $x \to +\infty$ must also be stated in order to eliminate this divergence. One more remark is in order: Inasmuch as the equations are "inviscid" in region A, there must be a thin wall layer in which the influence of viscosity is essential. However, the contribution of this layer to the induced pressure is $O(\mathrm{Re}^{-1/4}) \ll \Delta p$ [$\Delta p = O(u_w^2)$] and will therefore be disregarded from now on.

3. We now examine the case of small h in closer detail. The equations describing the flow in regions A and B are linearized everywhere with the possible exception of a critical layer in the vicinity of the point Y = 1. We consider region A first. The expansion $\tilde{U} = -1 + Y + hU^{(1)} + \ldots$, $\tilde{V} = hV^{(1)} + \ldots$, $\tilde{P} = hP^{(1)} + \ldots$ is valid outside the critical layer; substituting it in Eq. (2.2), we obtain

$$-1)\partial U^{(1)}/\partial X + V^{(1)} + P^{(1)\prime}(X) = 0,\ \partial U^{(1)}/\partial X + \partial V^{(1)}/\partial Y = 0; \quad (3.1)$$

$$V^{(1)}(X,\ 0) = 0,\ (\partial U^{(1)}/\partial X)(X,\ +\infty) = \theta(X) - P^{(1)}(X). \quad (3.2)$$

Linearization eliminates the longitudinal component of the velocity vector from the equations, leaving only its derivative with respect to X. Consequently, if one is concerned with only the behavior of the pressure in region A, it is not necessary to specify the initial velocity profile, which is determined by the matching condition (2.3). This kind of simplification is valid only in the linear problem. This is not the case in the nonlinear problem, and it is required to know the initial velocity profile, which is obtained by solving the problem for region B.

For an arbitrary function $P^{(1)}(X)$ the solutions of Eq. (3.1) subject to the impermeability condition and the condition (3.2) at the outer boundary do not coincide in general. They are written

$$\partial U^{(1)}/\partial X = P^{(1)\prime}(X),\ V^{(1)} = -YP^{(1)\prime}(X); \quad (3.3)$$

$$\partial U^{(1)}/\partial X = \theta(X) - P^{(1)}(X),\ V^{(1)} = -P^{(1)\prime}(X) + (Y-1) \times \\ \times (P^{(1)}(X) - \theta(X)). \quad (3.4)$$

Relation (3.3) satisfies the impermeability condition and, in any case, is valid for Y < 1; Eq. (3.4) satisfies the condition at the outer boundary and is valid for Y > 1. The only point (apart from the viscous wall layer) at which Eqs. (3.1) are no longer applicable is Y = 1. To match the solutions (3.3) and (3.4) at Y = 1, we introduce a new region (region 1 in Fig. 1) $Y - 1 = h^{1/2}\tilde{Y}$. The expansion $\tilde{U} = h^{1/2}\tilde{Y} + hU + \ldots$, $\tilde{V} = -hP^{(1)\prime}(X) + h^{3/2}V + \ldots$ holds in this region (which is usually called the critical layer). We note that "viscous" terms, which are $O(Re^{-1/2}u_w^{-4})$ in region A, now cannot be ignored in this layer. Substituting the resulting expansions in Eq. (2.2) and including "viscous" terms, we write

$$\tilde{Y}\partial U/\partial X + V - P^{(1)\prime}(X)\partial U/\partial\tilde{Y} = \mu\partial^2 U/\partial\tilde{Y}^2,\ \partial U/\partial X + \partial V/\partial\tilde{Y} = 0. \quad (3.5)$$

In the limit Y → ±∞ the boundary conditions acquire the form

$$\lim_{\tilde{Y}\to-\infty}\frac{\partial U}{\partial X} = P^{(1)\prime}(X),\quad \lim_{\tilde{Y}\to+\infty}\frac{\partial U}{\partial X} = \theta(X) - P^{(1)}(X).$$

Before stating the boundary conditions in the limit X → ±∞, we must make note of the fact that $\mu = (\mu_w/\mu_e)(\rho_e/\rho_w)^{-1/2}u'(0)(M_e^2 - 1)^{-1/2}Re^{-1/2}u_w^{-1}\Delta p^{-3/2}$ is the ratio of the length of the interaction zone to the "viscous" decay length for perturbations in the critical layer in region B. Also, since μ = O(1) in general, the boundary conditions with respect to the longitudinal coordinate are extinction conditions. To arrive at the final statement of the problem, we differentiate Eq. (3.5) with respect to Y and denote $\omega = \partial U/\partial\tilde{Y}$ as the vorticity in the critical layer. For the vorticity we have (omitting all indices)

$$Y\partial\omega/\partial X - P'(X)\partial\omega/\partial Y = \mu\partial^2\omega/\partial Y^2,$$

$$\frac{d}{dX}\int_{-\infty}^{+\infty}\omega\,dY + P(X) + P'(X) = \theta(X),\quad \lim_{Y\to\pm\infty}\omega = \lim_{X\to\pm\infty}\omega = 0.$$

We see at once that one of the solutions of the problem is ω = 0, $P(X) = \theta(x)[1 - e^{-X}]$, which coincides with the solution obtained by passing to the limit from the triple-decker model. However, the uniqueness of this solution has not been proved.

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REFINEMENT OF THE EQUAL-AREAS LAW FOR UNSTEADY PLANE SHOCK WAVES OF MODERATE INTENSITY

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1. The equal-areas law, or Whitham's law, is often used in the theory of weak shock waves; it states that the integral of the flow velocity (excess density or pressure) is time-invariant (see [1-3]):

$$\int_L v\,dX = \text{const}, \tag{1.1}$$

where L is the wavelength, and $X = x - c_0t$ is the accompanying coordinate. Equation (1.1) can be regarded as a conservation law for the simple-wave equation

$$v_t + \varepsilon v v_X = 0,$$

which is valid in the presence of discontinuities [1]. Here (and from now on) $\varepsilon = (1/2)(\gamma + 1)$. The equal-areas law describe the onset and development of an isentropic discontinuity in a simple wave and the laws governing the decay of plane shock waves.

Equation (1.1) follows from an analysis of the corresponding viscous problem. The simple-wave equation must be used with allowance for real dissipation (the Burgers equation in nonlinear acoustics) [2]:

$$v_t + \varepsilon v v_X - (b/2\rho_0)v_{XX} = 0 \tag{1.2}$$

[$b = \zeta + (4/3)\eta + (\gamma - 1)\kappa/c_p$ is the dissipation factor, η and ζ are the shear and bulk viscosity coefficients, and κ is the thermal conductivity]. An existing theorem [4] states that Eq. (1.2) has a unique conservation law, which coincides with Eq. (1.1).

We note that the Burgers equation is not a formal device in this case, but is derived from the complete system of equations of motion upon satisfaction of the natural asymptotic conditions [2, 5]

$$\partial/\partial X = O(1),\ \partial/\partial t = O(\mu);\ v,\ \rho' = O(\mu);\ \eta,\ \zeta,\ \varkappa = O(\mu), \tag{1.3}$$

where μ is a small parameter (the wave amplitude), and ρ' is the excess density. Conditions (1.3) have the physical significance that the traveling waveform varies slowly as a result of weak nonlinearity and dissipation.

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